B.Sc- part-I (H). paper-I Abstract Algebra.

20-07-2020

Q.1 Define Abelian group or Commutative group, non-abelian group or non-commutative group, finite group, infinite group and the order of a group.

Soln: Abelian OR Commutative group:

A group (G, o) is called an abelian or commutative if

$$a o b = b o a \quad , \forall\, a, b \in G$$

If the group G does not satisfy commutative axiom $a o b = b o a$, then G is called non-abelian or non-commutative.

The set of all even integers with zero forms an abelian group with respect to addition composition.

Finite Group:→ If a group contains a finite number of elements, it is called a finite group.

Infinite group:→ If the number of elements in a group is infinite, it is called an infinite group.

Order of a Group:→ The number of elements in a finite group is called the order of the group. It is denoted by $O(G)$.

An infinite group is called a group of infinite order.

Q.2. Show that the set Z of integers (+ve or -ve including 0) is an infinite abelian group with additive binary operation.

Soln; Let the group-axioms to all integers.

(I) Closure Property:- Closure property is satisfied because the sum of any two integers is an integers.

II Associativity: The ~~associativity~~ property is satisfied, because of a, b, c are any three integers, then

$$a+(b+c) = (a+b)+c$$

(III) Existence of identity:- The axiom on identity is satisfied, because 0 is the identity element in the set Z such that

$$a+0 = a \quad , \forall\, a \in Z$$

(IV) Existence of inverse:→ The axiom on inverse is ~~called~~ satisfied because the inverse of any integer a is the integer $-a$ such that

$a+(-a) = (-a)+a = 0$, the identity element.

(v) Commutativity: 20-07-2020

Since, we know that $a+b=b+a$, $\forall$ $a,b \in Z$, the commutative law is satisfied.

Also, the number of elements in Z is infinite. Hence, the set Z is an infinite abelian group with additive binary operation.

Proved.

Q.3. Show that the set $\{1,-1,i,-i\}$ is an abelian finite group of order 4 under multiplication.

Soln: Closure Property: Closure property is satisfied as

$1(-1)=-1$, $1\cdot i=i$, $i(-i)=1$, $i(-1)=-i$, etc.

(ii) Associativity: Associative property is satisfied as

$(1\cdot i)(-i)=1$, $\{i(-i)\}=1$, $\{1\cdot i\}\cdot(-i)=1$, $\{i(-1)\}=-i$, etc.

(iii) Existence of identity: Axiom on identity is satisfied, 1 being the multiplicative identity.

(iv) Existence of inverse:- Axiom on inverse is satisfied since since the inverse of each element of the set exists.

$1\cdot 1=e=1$, $(-1)(-1)=e=1$, $i(-i)=e=1$, $(-i)(i)=e=1$

(v) Commutativity. The commutative law is also satisfied as

$1(-1)=(-1)\cdot 1$, $(-1)i=i(-1)$, etc.

Since, there are four elements in the given set, hence it is a group of order 4.